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NEW EXTENSIONS OF SIX-PORT THEORY AND PRACTICE

L. Kaliouby, R.G. Bosisio

École Polytechnique of Montreal

Montréal, Québec, Canada

Abstract

This paper presents new extensions of six-port theory and practice from the following three points of view: 1) theory: a new relationship between reflection coefficient and calibration constants is given, 2) calibration: a new simple explicit six-port calibration solution using three positions of a sliding short is developped, 3) design: a new six-port design for continuous Γ-plane is introduced.

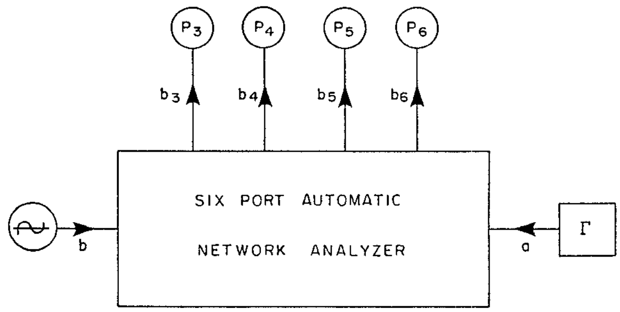

Figure 1: A six-port network analyzer measure reflection coefficient $\Gamma=a/b$ by means of four output power readings P_3, P_4, P_5, P_6.

Introduction

Six-port automatic network analyzers measure reflection coefficient Γ by means of four output power readings of the form: (Fig. 1).

$$P_3=|b_3|^2=|aA+bB|^2 \quad (1)$$
$$P_4=|b_4|^2=|aC+bD|^2 \quad (2)$$
$$P_5=|b_5|^2=|aE+bF|^2 \quad (3)$$
$$P_6=|b_6|^2=|aG+bH|^2 \quad (4)$$

where a and b represent respectively the reflected and incident waves, and A, B,... H are complex values function of six-port design (1,2,3).

It has been shown that if one of the power readings has to be proportional to the incident wave, that is, $P_4=K|b|^2$ with C=0, then the ideal six-port junction must have B/A, F/E, H/G equal in module with a phase difference of 120° (2). However, with existing standard microwave components, a 135-135-90 configuration is more easily realized (3) with $B/A=\sqrt{2}\angle 90$, $F/E=2\angle 225$, $H/G=2\angle 315$.

Moreover, in practice, experimental measurements show that the values of A, B,... H diverge from design objectives, and vary with frequency. Thus, for precise measurements of Γ, it's necessary to first calibrate the six-port, that is, to determine the values of A, B,... H as a function of frequency. Afterwards, the values of Γ can be calculated using the three power ratios P_3/P_4, P_5/P_4, P_6/P_4 of the form.

$$\frac{P_\eta}{P_4} = \left|\frac{b_\eta}{b_4}\right|^2 = |K_\eta|^2\left|\frac{\Gamma-q_\eta}{\Gamma-q_4}\right|^2 \quad (5)$$

with $K_3=A/C$, $q_3=-B/A$
$K_5=E/C$, $q_5=-F/E$
$K_6=G/C$, $q_6=-H/G$
$q_4=-D/C$

This paper presents new extensions of six-port theory and practice from the following three points of view:

. <u>Six-port theory</u>
In this paper, it is shown that the complex constants q_η and q_4 represent respectively the average value of Γ when

$$\frac{P_\eta}{P_4} < |K_\eta|^2 \text{ and } \frac{P_\eta}{P_4} > |K_\eta|^2.$$

The averaging is done with respect to the phase difference between signals b_η and b_4. This property provides an attractive geometrical interpretation and gives interesting insight into the six-port calibration and measurement process.

. Six-port calibration

The calibration of six-port consists in determining eleven constants composed of four complex values: B/A, D/C, F/E, H/G and three real ones: $|A/C|^2, |E/C|^2, |G/C|^2$. Various algorithms exist for such purposes (4,6,7). However, the objective still remains to obtain the simplest solution, explicit if possible, using the minimum number of loads. In this paper, a simple explicit solution for this problem is developped using only three positions of a sliding short, with a phase difference of 120°. It is shown that, in this case, the algorithm used to calculate the four complex calibration constants can be made very similar to the calculation of Γ for an ideal 120° junction.

. Six-port design

In experimental and laboratory work, it's often required to make precise real-time or swept frequency measurements for fine tuning purposes or verification of circuit characteristics within a given bandwidth. Recently, this has been made realisable by using a six-port chart that is calculated from the calibration constants (9,10). However, with existing six-port design, it's only possible to make readings in a distorted (almost tridimensional) Γ plane. In this paper, a new six-port design is developped such that real-time or swept frequency measurements can be made in the more conventional Γ plane, using only power ratio values.

A New Relationship Between Reflection Coefficient and Calibration constants

Let's first consider a simple case where $q_\eta=1$, $q_4=j$ and $K_\eta=1$.

Figure 2 shows locus of constant module α and phase θ of

$$\frac{b_\eta}{b_4} = \alpha \measuredangle \theta = \frac{aA+bB}{aC+bD} \quad \text{in the } \Gamma\text{-plane.}$$

Figure 3 shows for a circle with α=.2,.4,.6,.8, the x and y values of Γ as a function of θ. It is seen that the average value of x is 1, and the average value of y is 0, corresponding to the x and y coordinates of q_η. Similarily, for α<1, the average value of Γ is (0,1), that is, q_4.

More generally speaking, it can be shown that for any value of q_η, q_4, K_η, the relationship:

$$\Gamma_{av} = \frac{1}{2\pi}\int_0^{2\pi} (\Gamma x,\ \Gamma y)\ d\theta \qquad (6)$$

$$= q_\eta \text{ if } \frac{P_\eta}{P_4} < |K_\eta|^2$$

$$= q_4 \text{ if } \frac{P_\eta}{P_4} > |K_\eta|^2$$

still holds.

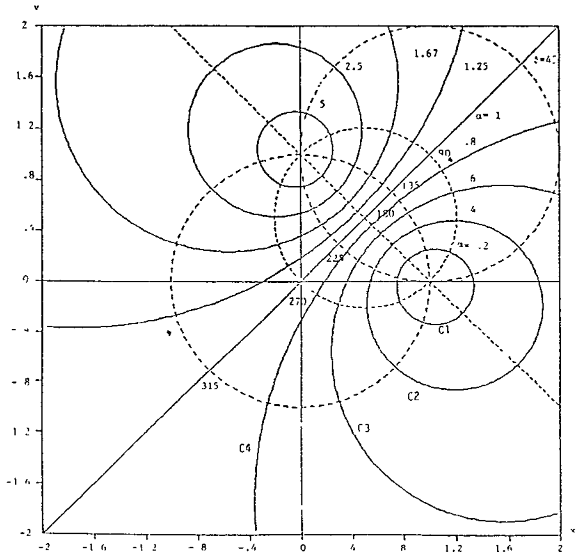

Figure 2: Locus of constant module, α, of

$$\frac{b_\eta}{b_4} = K_\eta \frac{\Gamma - q_\eta}{\Gamma - q_4}$$

and constant phase, θ, of b_η/b_4, in the Γ plane, for $q_\eta=1$, $q_4=j$, $K_\eta=1$.

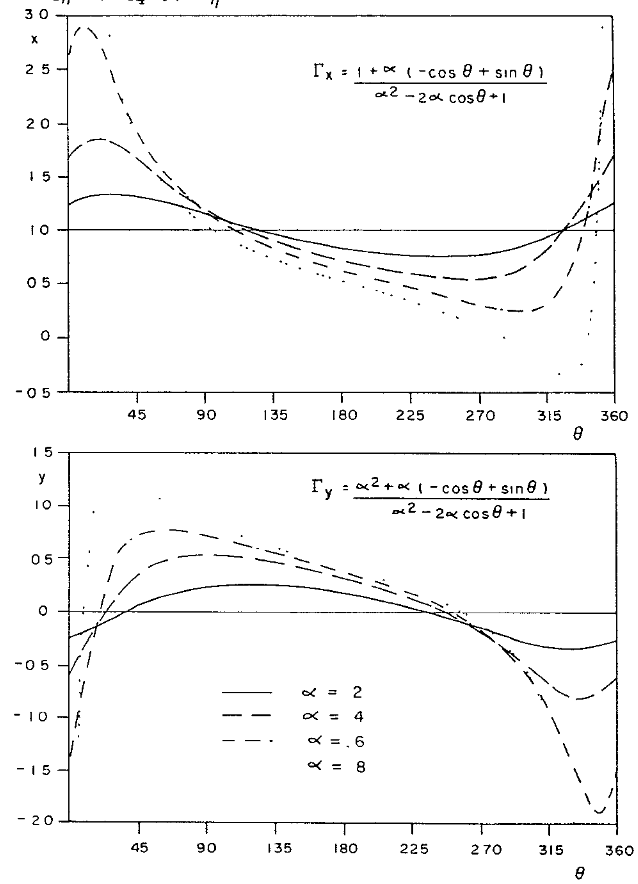

Figure 3: The x and y coordinates values of Γ, as a function of θ, for a circle with α=0.2, 0.4, 0.6, 0.8.

A Simple Explicit Six-Port Calibration Solution Using Three Positions of a Sliding Short

Let's designate by P_{33}, P_{35}, P_{36} the power output P_3 when the load is successively three positions of a sliding short with a phase difference of 120° difference; ($\emptyset_3=0$, $\emptyset_5=120$, $\emptyset_6=240$) and let's designate by P_{34} the power output P_3 when the load is a matched load. P_{34} may be calculated from P_{33}, P_{35}, P_{36})

We have then

$$\frac{P_{3\eta}}{P_{34}} = |(B/A)^{-1} + |\measuredangle{-\emptyset_\eta}|^2 \tag{7}$$

The value of $(B/A)^{-1}$ can then be simply calculated as the intersection of three circles in the $(B/A)^{-1}$ plane (Fig.4) or as

$$(B/A)^{-1} = \frac{\sum_{i=1}^{4} (\alpha_i+\beta_i)P_{3i}}{\sum_{i=1}^{4} \gamma_i P_{3i}} \tag{8}$$

Similar remark apply for the evaluation of F/E, H/G, D/C. It's only required afterwards to correct for the exact reference phane, by use of a fixed short or open.

The values of $|A/C|$, $|E/C|$, $|G/C|$ are finally evaluated using equation (5).

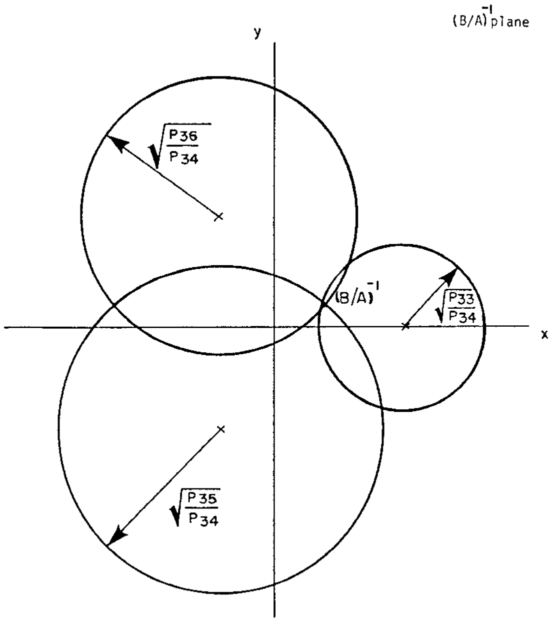

Figure 4: The four complex calibration constants can be simply calculated as the intersection of three circles, with center given by $1\measuredangle 0$, $1\measuredangle 120$, $1\measuredangle 240$. For example, the value of $(B/A)^{-1}$ can be calculated as the intersection, in the $(B/A)^{-1}$ plane, of three circles of radius given by $\sqrt{P_{33}/P_{34}}$, $\sqrt{P_{35}/P_{34}}$, $\sqrt{P_{36}/P_{34}}$.

A New Six-Port Design for Continuous Γ-plane Display

In order to be able to make readings in the conventional Γ-plane using only power ratio values, the mathematical objective design has to be:

$$\frac{P_3}{P_4} = \Gamma x + \text{constant} \tag{9}$$

$$\frac{P_5}{P_4} = \Gamma y + \text{constant} \tag{10}$$

From equation (5) this relationship implies:

$$k\left(\frac{m^2+m_3{}^2-2mm_3\cos(\varphi-\varphi_3)}{m^2+m_4{}^2-2mm_4\cos(\varphi-\varphi_4)}\right) = m\cos\varphi + \text{const.} \tag{11}$$

$$k\left(\frac{m^2+m_5{}^2-2mm_5\cos(\varphi-\varphi_5)}{m^2+m_4{}^2-2mm_4\cos(\varphi-\varphi_4)}\right) = m\sin\varphi + \text{const.} \tag{12}$$

where $\Gamma = m\measuredangle\varphi$

$B/A = m_3\measuredangle\varphi_3$, $F/E = m_5\measuredangle\varphi_5$, $D/C = m_4\measuredangle\varphi_4$

Using these equations, it can be shown that to satisfy these requirements in the region $|\Gamma|<|$, the optimal values of B/A, D/C, F/E are (Fig. 5).

$-B/A=2.5\measuredangle 190$, $-D/C=10\measuredangle 225$, $-F/E=2.5\measuredangle 260$

In fact, figure 6 shows that in this case, P_3/P_4 and P_5/P_4 becomes then practically linear function of Γ_x and Γ_y. Thus, the six-port chart, in the region $|\Gamma|<|$, becomes then practically a Γ-plane (Fig.7).

It becomes then extremely easy for a network analyzer user to make real-time or swept frequency measurements, within a minimum familiarization time with the system.

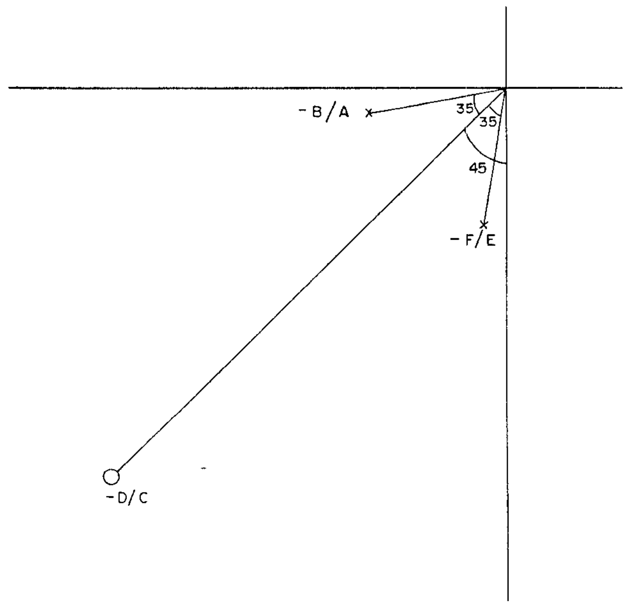

Figure 5: New six-port design with $-B/A=1.5\measuredangle 190$, $-D/C=10\measuredangle 225$, $-F/E=2.5\measuredangle 260$, that is, a 35-35 configuration.

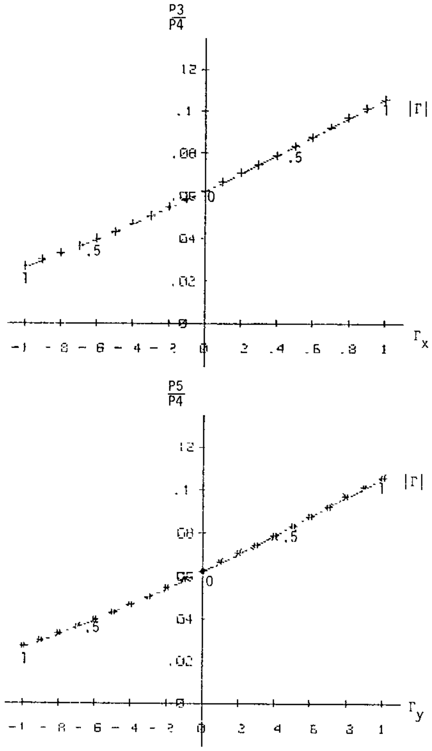

Figure 6.a): The relationship between power ratio P_3/P_4 and Γ_x is practically linear in the horizontal axis, that is for φ=0 and φ=180.

Figure 6b): The relationship between power ratio P_5/P_4 and Γ_y is practically linear in the vertical axis, that is, for φ=90 and φ=270.

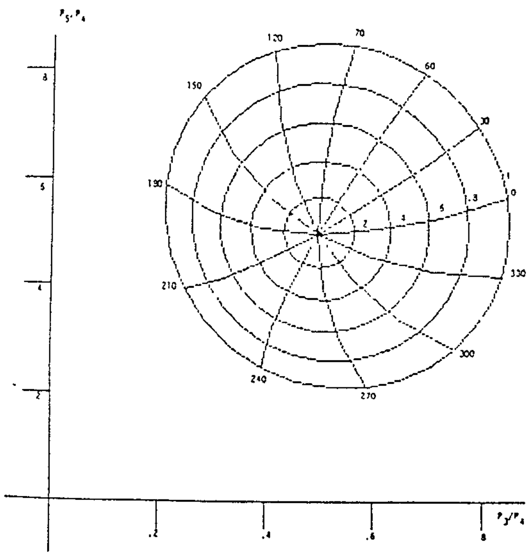

Figure 7: The six-port chart for a six-port junction characterized by -B/A=2.5∠190, -D/C=10∠225, -F/E=2.5∠260 is practically a Γ plane for $|\Gamma|<1$.

Experimental results

An experimental set-up has been developped for the measurement of reflection coefficient using six-port concept.

The precision of calibration algorithm using three positions of a sliding short is 1% in module and 2° in phase.

The frequency where the design is the closest to the optimum is at 3.4 GHz.

Conclusion

This paper has presented new extensions of six-port theory and practice from the following three points of view: 1) theory: a new relationship between reflection coefficient and calibration constants has been given, 2) calibration: a new simple explicit six-port calibration solution using three positions of a sliding short has been developped 3) design: a new six-port design for continuous Γ-plane display has been introduced.

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